NOTATION

$\Sigma P'$, sum of external forces acting on a particle which are not related to its displacement; F, resisting force of carrying agent of aerosol; m, ρ_1, d, V, and W, mass, density, diameter, absolute and relative velocities of particle; d_e, equivalent diameter of particle; t, time; U, ρ, μ, ν, velocity, density, absolute and kinematic viscosities of carrying agent of aerosol; $\pi = 3.14$, ratio of circumference to diameter; ψ, drag coefficient of spherical dust particles; $Re = Wd/\nu$, Reynolds number for particle; A and n, coefficients (Table 1); k, shape factor of particle; $X = \Sigma F'/m$, acceleration produced by forces on particle which are not related to its displacement; e, base of natural logarithms; W_c, rate of settling of particles; T, absolute temperature; n_1, exponent; b, coefficient depending on rate of heat exchange of gas with channel walls and suspended particles; 0, subscript indicating that quantity refers to conditions at entrance to section under consideration; D, diameter of apparatus; α_T, heat-transfer coefficient; ΔT, temperature head; ω, water equivalent of flow; x, y, running coordinates; g, acceleration due to gravity; $c = 1 - \rho/\rho_1$, coefficient taking account of buoyancy of gas.

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GRAVITATIONAL SEPARATION OF DISPERSE SYSTEMS WITH PRELIMINARY AGGREGATION OF PARTICLES IN AN ELECTRIC FIELD

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The separation of a disperse medium in a gravitational sedimentation tank is investigated. An external electric field is used for preliminary aggregation of the conducting disperse phase.

At present, the separation of disperse media (for example, the removal of oil from water) is carried out by gravitational precipitation of the disperse-phase particles in a sedimentation apparatus, with preliminary aggregation of the particles. If the particles are conducting, more intense aggregation (coalescence) will occur when an external electric field is applied. A coalescence unit is an apparatus for the preliminary aggregation of emulsions in a constant electric field. In spite of the wide use of sedimentation apparatus, there has been little study of its efficiency.

All the existing types of sedimentation tanks may be divided into two classes: those with vertical and horizontal fluxes in the sedimentation region. One of the main characteristics of these units determining the efficiency of separation of disperse media is the ratio between the concentrations of the disperse phase at the outlet and inlet of the sedimentation tank

$$\lambda = \frac{W_2}{W_1}. \tag{1}$$

It is possible to establish which of the two types of sedimentation tanks is the more efficient for the separation of disperse media from a comparison of the respective values of λ. The more efficient apparatus will have the lower value of λ.

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Suppose that at the inlet of the sedimentation tank there is a disperse phase with a given distribution of disperse-phase particles over the radius, $n_1(r)$. Then, assuming that the bulk concentration of the particles is small, they move independently of one another and do not coalesce. At the outlet from the sedimentation tank, the particle distribution over the radius is

$$n_2(r) = k(r)\, n_1(r), \tag{2}$$

where the coefficient $k(r)$, characterizing the efficiency of particle separation, is independent of the velocity profile at the inlet if the flow is laminar and there are no stagnant regions within the tank. For a vertical sedimentation tank [1]

$$k(r) = \begin{cases} 1, & r < r_0, \\ 0, & r \geqslant r_0, \end{cases} \tag{3}$$

$$r_0 = (9HQ\mu/2\Delta\rho g V)^{1/2},$$

while for a horizontal tank

$$k(r) = \begin{cases} 1 - \left(\dfrac{r}{r_0}\right)^2, & r < r_0, \\ 0, & r \geqslant r_0. \end{cases} \tag{4}$$

If $n(r, t)$ is replaced by the density of the distribution $w(r, t)$ of the disperse-phase volume over the radius, Eqs. (1)-(4) lead to the following results: for a vertical sedimentation tank

$$\lambda_V = \int_0^{r_0} w_1(r, t)\, dr \Big/ \int_0^{\infty} w_1(r, t)\, dr \tag{5}$$

and for a horizontal tank

$$\lambda_g = \lambda_V - \int_0^{r_0} r^2 w_1(r, t)\, dr / r_0^2 \int_0^{\infty} w_1(r, t)\, dr. \tag{6}$$

Thus, to determine the coefficients λ_V and λ_g it is necessary to know the density of the distribution $w_1(r, t)$ of disperse-phase particles in the emulsion arriving at the inlet of the sedimentation apparatus.

Since the emulsion is subjected to preliminary aggregation in an electrical coalescence unit before it reaches the sedimentation tank, $w_1(r, t)$ may be regarded as the distribution obtained at the outlet from the coalescence unit. Hence, $w_1(r, t)$ will depend on the distribution at the inlet to the coalescence unit and on the residence time in the electric field of the coalescence unit.

Consider now the determination of these dependences. Assume that the process of coalescence in the electric field is spatially homogeneous; this is the case, for example, in the agitation of an emulsion. Note that intense agitation may result in turbulization of the flow and crushing of the disperse-phase particles, which is not considered in the present model.

In the spatially homogeneous case the change with time in the density of the particle distribution over the volume as a result of coalescence may be described by the relation

$$\frac{\partial n(v, t)}{\partial t} = \frac{1}{2} \int_0^{v} K(\theta, v-\theta)\, n(\theta, t)\, n(v-\theta, t)\, d\theta - n(v, t) \int_0^{\infty} K(v, \theta)\, n(\theta, t)\, d\theta,$$
$$n(v, 0) = n_0(v). \tag{7}$$

Here $K(v, \theta)$ is a coefficient determining the collision probability for drops of volumes v and θ; it depends on the mechanism of the drop interactions.

It is necessary to add to Eq. (7) the conservation condition for the bulk concentration of the disperse phase

$$\int_0^\infty \theta n(\theta, t)\, d\theta = \int_0^\infty \theta n_0(v)\, d\theta \equiv c, \tag{8}$$

which is obtained from Eq. (7) when the left-hand and right-hand sides are integrated with respect to the volume v between 0 and ∞.

The numerous investigations of Eq. (7) [2-7] have resulted in several accurate solutions for particular cases when the coalescence kernel is constant or linearly dependent on the volumes v and θ, and an approximate solution of Eq. (7) has been found by the method of moments. For a kernel of more complex structure (a nonlinear dependence on v and θ) and for initial distributions that are close to real conditions, it has not been possible to obtain an analytic solution. Numerical methods have been developed for the solution of Eq. (7) in these cases [8-10] and, using these methods, solutions have been obtained either for kernels of the above types (for the comparison of the numerical and accurate solutions) or for kernels that are of limited application in considering the coalescence of emulsions.

This problem was discussed in detail in [12] and the coalescence kernel K(v, θ) was determined, taking the hydrodynamic and electric forces of drop interaction fully into account (only pair interactions were considered). The result obtained for K(v, θ) was

$$K(v, \theta) = p\sigma_1(v, \theta)\,|V_v - V_\theta|; \tag{9}$$

$$\sigma_1 = 0.57\pi b^2 S\,[1 - \exp(5.75\,k_1)];$$

$$k_1 = a/b; \quad a = \min(r_v, r_\theta); \quad b = \max(r_v, r_\theta);$$

$$r_v = (3v/4\pi)^{1/3}; \quad r_\theta = (3\theta/4\pi)^{1/3};$$

$$V_\alpha = \alpha r_v^2; \quad V_\theta = \alpha r_\theta^2; \quad \alpha = 2\Delta\rho g/9\mu,$$

$$S = 3\varepsilon E^2 k_1^2/4\pi\Delta\rho g a\,(1 - k_1^2).$$

Here p is the probability of drop coalescence, assumed constant.

The initial distribution $n_0(v)$ is taken in the form of a logarithmic normal distribution

$$n_0(v) = \frac{v^*}{\sigma(v/v_m)}\exp\left(-\frac{\ln^2(v/v_m)}{2\sigma^2}\right), \tag{10}$$

which, as numerous experiments have shown, is found to apply to emulsions.

The parameter v_m in Eq. (10) is related to the mean value <v> and the dispersion σ by the expression $v_m = \langle v\rangle \exp(-\sigma^2/2)$; n* is chosen in accordance with Eq. (8).

Passing to the dimensionless variables

$$v = \frac{v}{v_m}, \quad n = \frac{nv_m^2}{c}, \quad \tau = \frac{0.024 p\varepsilon E^2 ct}{\pi\mu}, \quad R = \frac{r}{r_m}$$

in Eqs. (7)-(10) leaves Eq. (7) unchanged, while Eq. (8) takes the form

$$\int_0^\infty \theta n(\theta, \tau)\, d\theta = 1. \tag{11}$$

A detailed analysis of the solution of Eq. (7) with the kernel in Eq. (9) was given in [13] for various initial conditions.

By numerical integration (using the method of iteration) of Eqs. (7)-(11) with the initial condition in Eq. (10) it is possible to determine n(R, τ), w(R, τ), and also the integral characteristics

$$N = \int_0^\infty n(R, \tau)\, dR; \quad I = \int_0^{R_0} w(R, \tau)\, dR \equiv \lambda_v.$$

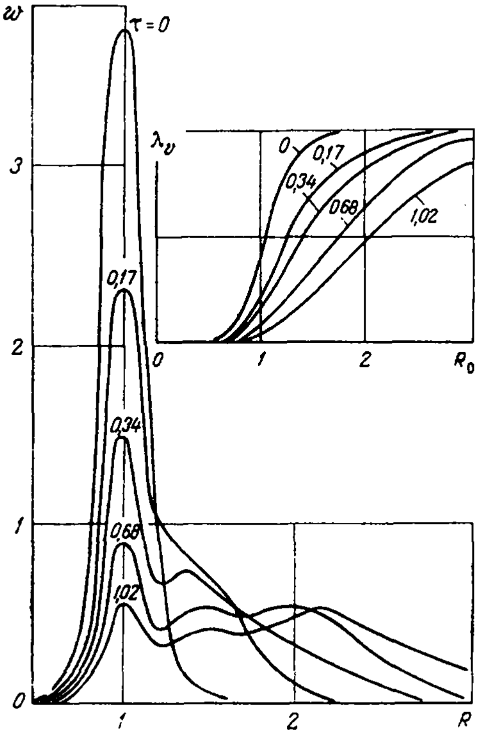

Fig. 1. Change in the particle distribution over the radius with σ = 0.3.

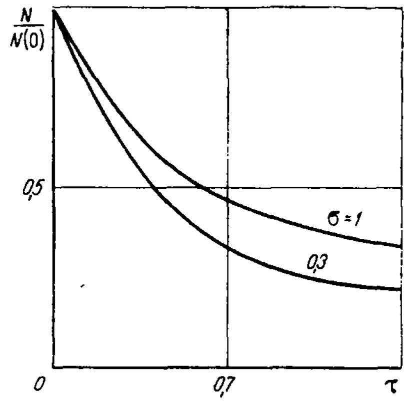

Fig. 2. Variation in the number of particles.

Distributions of w and I for σ = 0.3 are shown in Fig. 1. Note that there are two or more humps in the distribution of w. The number of humps increases with decrease in σ. For $\sigma \gtrsim 1$ the effect is not observed.

As time passes, the rate of coalescence drops rapidly as a result of the considerable reduction in the number of particles N (Fig. 2). Note that some time T after the beginning of coalescence the distributions of w for different initial values of σ are practically the same. The value of T depends on σ: With decrease in σ, T increases.

The curves in Fig. 3 show the dependences of the main characteristics of the horizontal λ_g and vertical λ_v (dashed line) sedimentation tanks obtained from Eqs. (5) and (6) for σ = 0.5.

Numerical solution of Eq. (7) is too time-consuming and inconvenient for actual calculations. It was shown in [13] that the approximate solution of Eq. (7) by the method of moments is in satisfactory agreement with the numerical solution. The approximate solution for the initial distribution

$$n_0(v) = \frac{N^2(0)}{c} \cdot \frac{\nu+1}{\nu!}\left(\frac{v}{v_m(0)}\right)^{\nu} \exp\left(-\frac{v}{v_m(0)}\right) \tag{12}$$

is

$$\begin{gathered} n(v, \tau) = \frac{N^2}{c} \cdot \frac{\nu+1}{\nu!}\left(\frac{v}{v_m}\right)^{\nu} \exp\left(-\frac{v}{v_m}\right), \\ N = N(0)\exp(-\beta\tau), \quad v_m = \frac{\langle v(0) \rangle}{\nu+1}\exp(\beta\tau). \end{gathered} \tag{13}$$

Here <v(0)> is the mean initial volume of the drop; ν is a parameter related to the dispersion σ(0) and with <v(0)> by the expression $\sigma(0) = (\nu + 1)^{-1/2}\langle v(0)\rangle$; β is a parameter dependent on ν (this dependence is given in [13]).

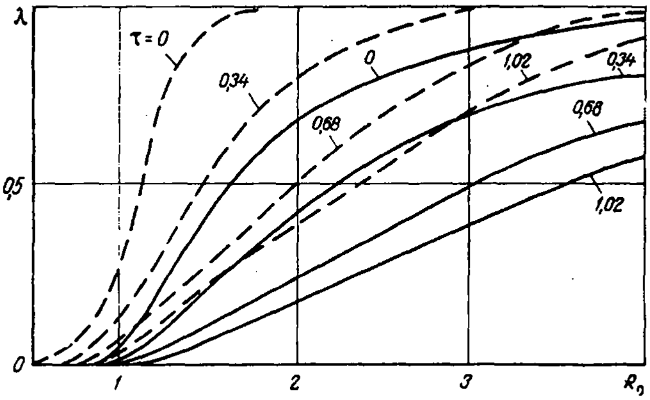

Fig. 3. Dependence of sedimentation-tank characteristics λ_g and λ_v on R_0 and τ.

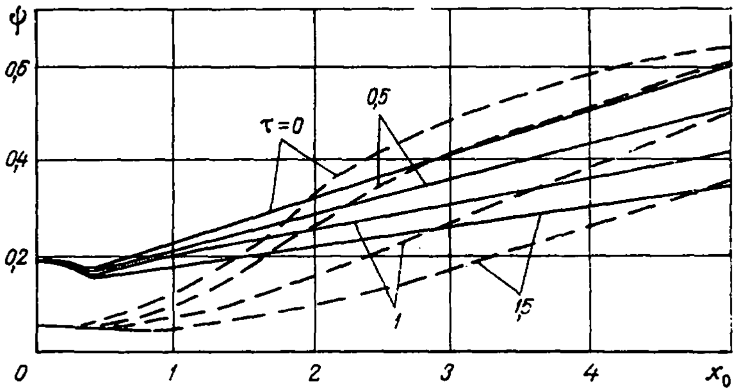

Fig. 4. Comparison of separation efficiency of disperse media by horizontal and vertical sedimentation tanks.

Note that with suitable choices of the parameters ν and $v_m(0)$ Eq. (12) is a good approximation to Eq. (10).

Using Eq. (13) the main characteristics λ_g and λ_v may be determined

$$\begin{gathered}\lambda_v = \frac{\gamma(\nu+2,\ x)}{(\nu+1)!}, \\ \lambda_g = \lambda_v - \frac{\gamma(\nu+8/3,\ x)}{(\nu+1)!\ x^{2/3}}, \\ x = x_0(\nu+1)\exp(-\beta\tau), \\ x_0 = v_0/\langle v(0)\rangle, \quad v_0 = 4\pi r_0^3/3.\end{gathered} \tag{14}$$

Here $\gamma(x, y)$ is an incomplete gamma function.

Using Eqs. (13) and (14) it is possible, given the parameters of the distribution at the inlet (the dispersion and the mean volume and bulk concentration of the disperse phase), to determine the analogous parameters at the outlet of the sedimentation tank.

It is of particular interest to compare the efficiency of separation for sedimentation tanks of the two types. Let $\psi = \lambda_g/\lambda_v$.

For the same bulk concentration of the disperse phase at the inlet, ψ is equal to the ratio of the corresponding bulk concentrations at the outlet of horizontal and vertical sedimentation tanks. The dependence $\psi(\tau, x_0)$ is shown in Fig. 4 for $\nu = 2$ and $\nu = 15$ (dashed curves). The difference between λ_g and λ_v is greatest when $x_0 \leqslant 1$. Since $x_0 \sim Q^{3/2}$, this region corresponds to low flow rates. With increase in x_0 (for example, increase in flow rate), the difference between λ_g and λ_v decreases, as $x_0 \to \infty$, $\psi \to 1$. Comparison of corresponding values of ψ for $\nu = 2$ and $\nu = 15$ — for these values, $\sigma(0)/\langle v(0)\rangle$ is equal to 0.58 and 0.25, respectively — shows that the parameter ψ is sensitive to the dispersion of the distribution at the sedimentation-tank inlet. Note that there is a minimum in the region $x_0 < 1$. With decrease in ν the minimum is expressed more sharply.

For each sedimentation tank of given geometric dimensions and with given flow rate and flow properties of the emulsion, there is a value $r_0 = (9HQ\mu/2\Delta\rho gV)^{1/2}$ such that after the residence time of the emulsion in the apparatus all drops of radius $r > r_0$ at the inlet are deposited. As follows from Eqs. (3) and (4), drops with $r < r_0$ remain in the flow of the emulsion for a vertical sedimentation tank and are deposited in a horizontal tank.

The dimensionless radius $R_0 = r_0/r_m(0)$ is plotted along the abscissa of Fig. 3. Therefore, the dependence shown in Fig. 3 may be considered as the dependence of λ on the geometric dimensions of the sedimentation tank, on the flow rate and properties of the emulsion, and also on the dimensionless residence time τ of the emulsion in the electrical coalescence unit.

Thus, the solution obtained can be used to find the residence time of the emulsion in the electrical coalescence unit necessary for aggregation of the disperse phase to a given size, thereby permitting significantly more rapid separation of the disperse media in gravitational sedimentation tanks. In addition, using the approximate solution in Eqs. (13) and (14) it is relatively simple to determine the sedimentation-tank characteristics for a given disperse-phase distribution at the tank inlet.

NOTATION

n, n_0, n_1, n_2, size distributions of particles; w, w_1, distributions of disperse-phase volume in terms of particle radius; v, particle volume; a, b, r, r_0, particle radii; R, dimensionless radius; H, V, height and volume of sedimentation tank; Q, flow rate of emulsion in sedimentation tank; μ, viscosity; $\Delta\rho$, density difference between continuous and disperse phases; g, acceleration due to gravity; $K(v, \theta)$, coalescence kernel; V_v, V_0, Stokes sedimentation rate of particles of volume v and θ; $\sigma_1(v, \theta)$, capture cross section; $k_1 = a/b$; $S = 3\varepsilon E^2 k_1^2/4\pi\Delta\rho g a(1 - k_1^2)$; p, drop-coalescence probability; ε, dielectric constant; E, external electric field strength; W_1, W_2, disperse-phase concentration at inlet and outlet of sedimentation tank; λ_g, λ_v, characteristics of horizontal and vertical sedimentation tanks; τ, dimensionless time; N, number of particles in unit volume of emulsion; σ, dispersion; $\langle v\rangle$, mean volume; ν, parameter of distribution; $\psi = \lambda_g/\lambda_v$.

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